

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH 2058 Honours Mathematical Analysis I 2022-23**  
**Homework 6**  
**2nd November 2022**

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Thursday. Additional announcement will be made if there are no homework that week.
  - Please send an email to [echlam@math.cuhk.edu.hk](mailto:echlam@math.cuhk.edu.hk) if you have any questions.
1. (P.116 Q15) Let  $A \subset \mathbb{R}$ ,  $f : A \rightarrow \mathbb{R}$ , and let  $c \in \mathbb{R}$  be a limit point of  $A$ . Suppose that  $f(x) \geq 0$  for all  $x \in A$ , so that the function  $\sqrt{f}$  is well-defined. Prove that if  $\lim_{x \rightarrow c} f$  exists, then we have  $\lim_{x \rightarrow c} \sqrt{f} = \sqrt{\lim_{x \rightarrow c} f}$ .
  2. (P.123 Q3) Define  $f(x) = |x|^{-\frac{1}{2}}$  for  $x \neq 0$ , show that  $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = +\infty$ .
  3. (P.123 Q11) Suppose that  $\lim_{x \rightarrow c} f(x) = L$  and  $\lim_{x \rightarrow c} g(x) = \infty$ , show that  $\lim_{x \rightarrow c} f(x)g(x) = \infty$  if  $L > 0$ . And provide a counter-example to in the case when  $L = 0$ .